Linear Dynamics for Everyone: Part 1

> Why natural frequency analysis is good for you and your design.

BY GEORGE LAIRD

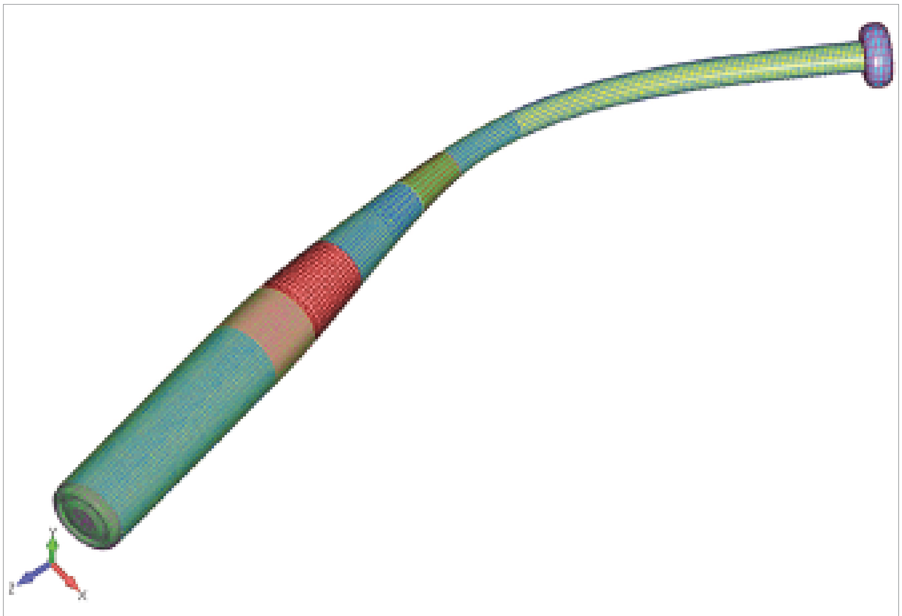

Figure 1: First vibration mode shape for an NCAA aluminum baseball bat is shown here.

Analysis work is rarely done because we have spare time or are just curious about the mechanical behavior of a part or system. It's typically performed because we are worried that the design might fail in a costly or dangerous manner. Depending on the potential failure mode our anxiety might not be too high, but given today's demanding OEMs and litigious public, the task could involve high drama with your name written all over it.

If you've done analysis, you're comfortable with the concepts involved in static stress analysis; you define the loading and boundary conditions, and identify success with a model bathed in soothing tones of gray and blue with nary a red region to be seen. However, in the back of your mind you might wonder about that large vibrating motor or the plant machinery that hums at a constant 12.5Hz. Alternatively, maybe you have an electronics enclosure that is to be mounted on the side of a building in an earthquake-prone region and your boss is questioning your bracket design. Whatever the case, you have the static world under control. What about the rest?

In this series of articles, we'll briefly review dynamic analysis fundamentals and see how they can easily be applied to make sure your design remains strong and rock solid in the face of dynamic events, whether simple vibrations, earthquakes, or even rocket launches.

KEEPING IT SIMPLE

Static stress analysis is the proverbial "walk-in-the-park" for most people doing analysis work. It feels straightforward: we apply a fixed load and examine the re-

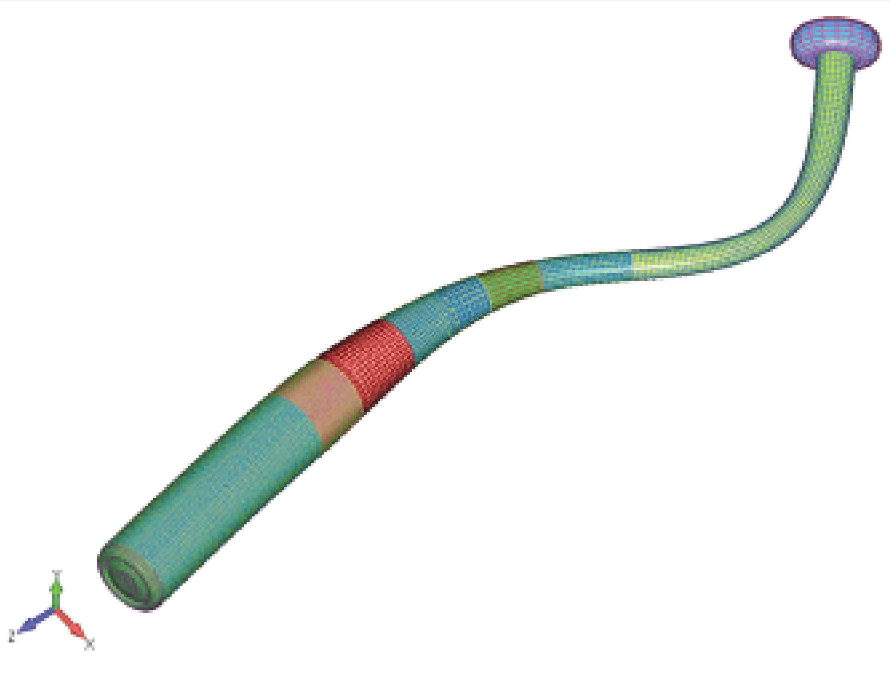

Figure 2: Second vibration mode shape is shown here for an NCAA aluminum baseball bat.

sulting static behavior (generally linear, given linear material behavior). We get back some nice clean stresses and deflections that hopefully match our intuition for how our design should behave. While there might be a few hiccups along the way, the end result usually appears logical to our mechanical minds.

The dynamic behavior of a structure can also be viewed in the same light if we just shift our perspective a bit and think in terms of how our structure should naturally deform during a dynamic event. Whenever a structure is hit or given some sort of time-varying load (transient or steady-state), it will respond to this load with a very characteristic behavior. If the load is not incredibly massive and the structure doesn't blow up or plastically deform as a result, then the dynamic response of your structure will most likely be linear. That is to say, if the load is removed and the structure is given a chance to calm down, then it will return to its undeformed state. This is the same concept to use in linear static stress analysis: when the load is removed the stress in the structure goes back to zero.

What exactly do we mean by characteristic dynamic behavior? All structures have natural or characteristic modes of vibration. The sound or note from a guitar string is all about its natural frequency of vibration. When a guitar string is plucked it will vibrate at a certain note or tone. This note is at the string's characteristic frequency.

Another example is aluminum baseball bats. The best aluminum baseball bats are designed with characteristic vibrations that attempt to limit the sting that occurs when you hit a ball outside the sweet spot on the bat. Each frequency creates a physical deformation or shape, and the total dynamic response of the bat is a combination of all its characteristic mode shapes (*see Figures 1 and 2*).

In finite element analysis (FEA), these natural frequencies are called eigenvalues and their shapes are noted as eigenvectors or eigenmodes. This nomenclature is rooted in German and the word eigen denotes "characteristic" or "peculiar to" and came into common use with mid-19th century mathematicians. With dynamic analyses, you'll also see the terms normal modes and normal modes analysis. The use of the word normal prior to mode is just another way to say natural, characteristic, or eigen. When describing mode shapes, our preference is to just say normal modes since they represent the inherent natural response of the structure.

A BEAM AS ONE EXAMPLE

If we picture a simply supported beam (fixed at one end), its natural mode shapes are determined by its geometry while its frequency of motion is fixed by its stiffness and density. Got all of that? Take a look at the graphic of our beam for its first three modes (*see Figures 3 and 4*). The first three modes of the beam are well-defined but come in pairs to cover all permissible ranges of motion for that beam. In 3D, the first mode can oscillate within a 360-degree envelope around its longitudinal axis. Numerically, the eigen solution process just gives us the two orthogonal modes, but it implies the full 360-degree envelope.

All structures have a nearly infinite number of permissible shapes or eigenvalues/eigenmodes. Fortunately, only the

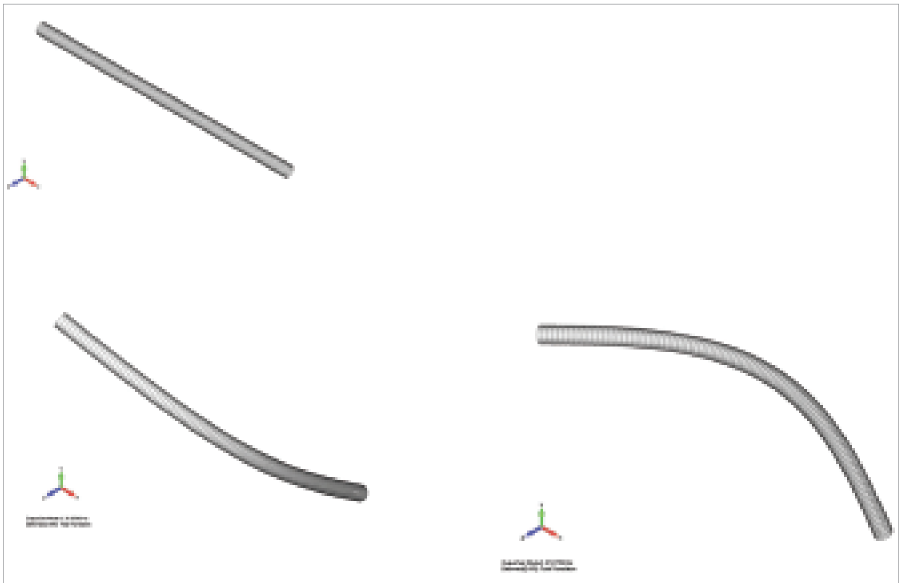

Figure 3: Undisturbed simple beam, plus two of the first vibration mode shapes (two directions of motion).

lower frequencies dominate the response of the structure so we can typically ignore the higher frequencies. A rule of thumb is that the first three modes capture the majority of the response of the structure and therefore one can safely ignore the higher frequencies. (The reasoning for this statement will be given in Part II of this series).

The frequency of these modes or their eigenvalues is dependent upon the stiffness and the density of the beam. The frequency equation for structures can thus be written as:

$$\omega = \sqrt{K/m}$$

where K is the stiffness of the structure and m is the mass. This wonderfully simple equation represents a great deal of information about the system. The classic way to graphically describe this equation is with a mass suspended by a spring, where the mass block can only move up and down or has one degree of freedom (DOF) in FEA parlance. The eigenmode of this system is up and down.

PAPER MILL DESIGN EXAMPLE

In commercially interesting structures, the same equation holds. The eigenvalue of the structure is still determined by

$$\omega = \sqrt{K/m}$$

For example, consider a forming board used within a paper mill. The structure is 10 meters long and made of stainless steel. The paper mill has an operating frequency of around 9Hz. If the structure's natural frequency is near this operating frequency, it will quickly resonate and tear itself apart. More importantly, it will also take the multi-million dollar paper mill along with it (*see Figures 5 and 6*).

The parameters of the original design placed the first mode at 8.4Hz, which would have been a disaster. The forming board is manufactured from 9.5mm-thick stainless-steel plates, so our first design inclination was to simply increase the thickness of the plates. We pursued this approach for several days but as we increased the thickness, the mass of the structure also increased almost in lockstep with the stiffness (see above equation). At the end of all this head banging, we got a marginal improvement (~11Hz resonance) with 25mm-thick plates, but it was going to cost a fortune to manufacture.

At this point we stepped back from our rush to find a solution and thought about how stiffness is developed in long slender structures. We realized that we had very little shear transfer between the top and bottom surfaces of the forming board. This insight led us to add diagonal steel rods that would connect the top and bottom planes and allowed us to keep the thickness of the plates at 9.5mm. The new design tested out on the computer with a first mode frequency of 13Hz. With the eigenvalue of the forming board now significantly higher than the operating frequency of the mill, resonance is impossible and the system is dynamically stable. Additionally, the thinner plates (9.5mm instead of 25mm) meant it was

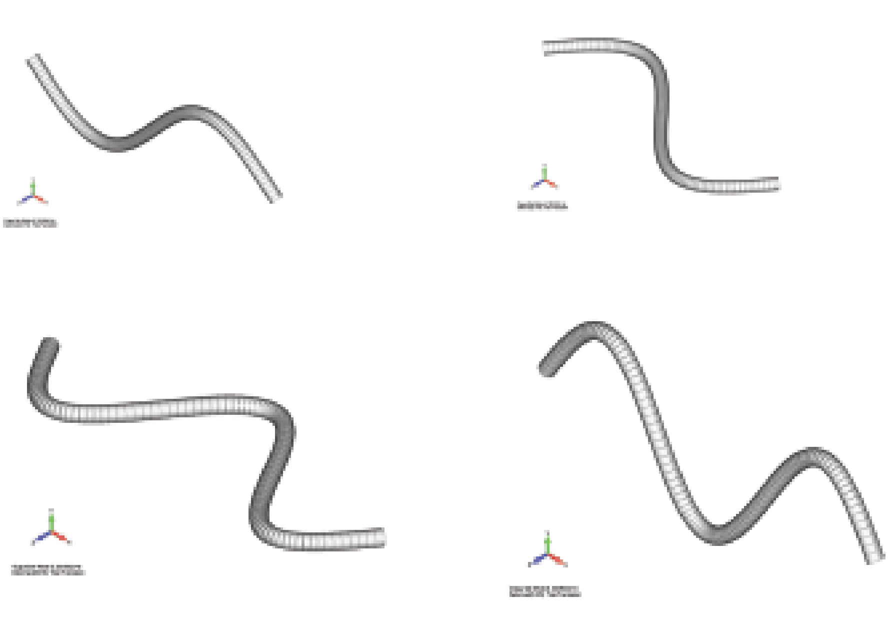

Figure 4: Second and third vibration modal-shape pairs for a simple supported beam.

more than half the cost of the first, marginal redesign.

DYNAMIC LOAD CONSIDERATIONS

When a structure is loaded in a transient or time-varying fashion (e.g., when an electric motor creates a constant, sinusoidally varying load), if the eigenvalue of the structure is lower or higher than the excitation frequency, the structure will just behave as if the load was applied statically. Let us say that we have this structure with an eigenvalue at 10Hz and it is whacked by a transient (e.g., half sine-wave with frequency of 10Hz), we would expect the structure to vibrate subsequent to the hit and then gradually return to its static zero-stress condition.

However, if the structure's dynamic load is time-varying (e.g., sine wave at 10Hz), the structure will resonate. If little damping is present (think metal or stiff plastic structures), then we may see the classic harmonic resonance that caused the collapse of the Tacoma Narrows Bridge in 1940. What kills structures is resonance, and the worst kind of resonance occurs when the structure sees the excitation load over and over again. The most effective way to eliminate this worry is to design your structure to have lower or higher natural frequencies than its operational frequency; this goal is the dominant reason for performing an eigen analysis.

THE FINAL MATH

In our prior discussion we haven't mentioned anything about the magnitude of an eigenmode. That is to say, we have discussed its frequency and its shape but left out any description of its magnitude. In eigen analysis (normal mode analysis) no load is applied to the structure. Without a load (e.g., a force or pressure), a prediction of the actual eigenmode is impossible. The extraction of the eigenmode (the shape of the permissible deformation mode) involves a fancy piece of math that is commonly available in a multitude of textbooks. The core thought is that we are solving the dynamic equation:

$$\{f(t)\} = [m]\{x''(t)\} + [C]\{x'(t)\} + [K]\{x(t)\}$$

If damping [C] is ignored (a good assumption for a lot of designs) and the applied force $f(t)$ is set to 0.0, the equation reduces to this more manageable formula:

$$[m]\{x''(t)\} + [K]\{x(t)\} = 0$$

This is the key equation for eigen analysis and states that only the mass and the stiffness of the structure control its natural modes.

To solve this equation see your favorite math handbook. The gist of the discussion is that the eigenvalue of the structure boils down into this elegant formula:

$$\omega = \sqrt{K/m}$$

And since no forces are used in the calculation of the eigenvalue, its associated eigenmode is dimensionless. Your FEA program then scales the eigenmode such that the maximum displacement within each mode shape is near 1.0 or some relative value tied to the mass of the structure. When these eigenmodes are dis-

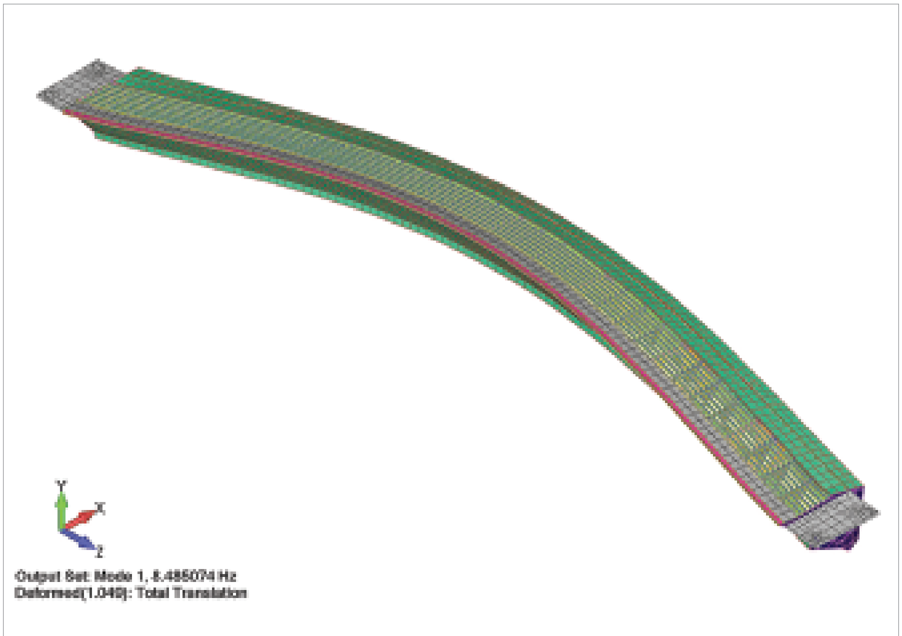

Figure 5: Mode 1 of vibrating paper-mill forming board

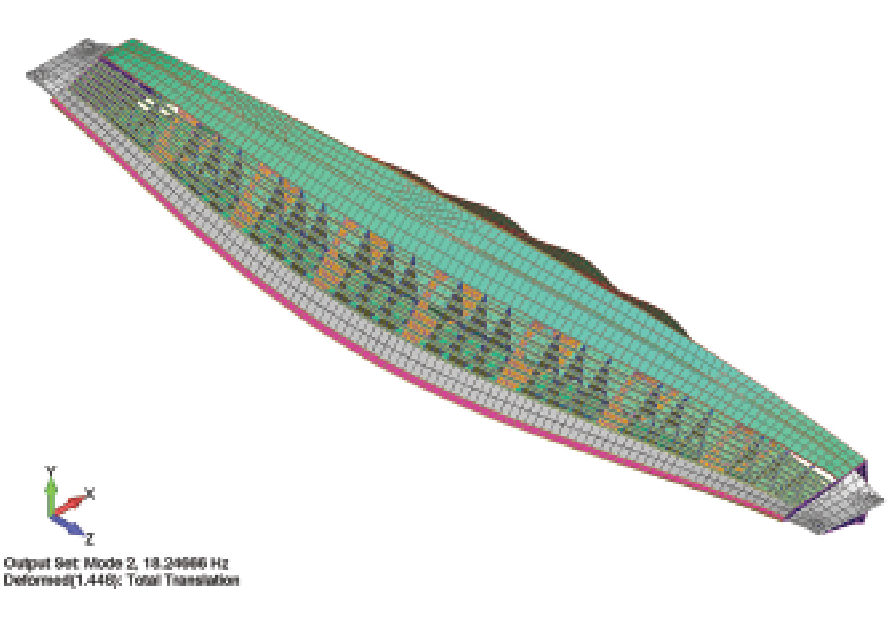

Figure 6: Mode 2 of vibrating paper-mill forming board

played within an FEA program, we see an imaginary magnitude; this visual can be problematic for many initiates who are first venturing into the eigen world of dynamic analysis, but we will discuss the implications in Part II of this series.

MODE ANALYSIS ESSENTIAL CHECKLIST

Determine what type of loading you may have on your structure and whether or not that loading might set up a resonant condition. Try to determine your loading frequencies and ensure that they fall outside of the eigenvalues of your structure.

Run an eigen analysis and look at the first three normal mode frequencies. See if they fall within your danger zone.

If the normal mode frequencies are outside your loading frequencies then stop. You are done and all is good.

If your normal mode frequencies are within your range of interest and you can't redesign around them, then stay tuned for our future articles. We will show that maybe it isn't that bad after all. ■

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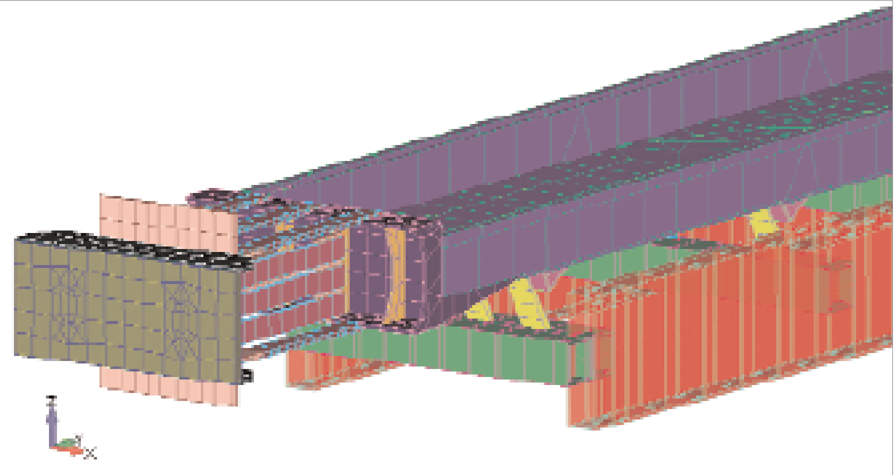
Linear Dynamics for Everyone: Part 2

> Vibration analysis can show detailed structural behavior under dynamic loading.

BY GEORGE LAIRD



The motor mount for this vibrating conveyor is the mass of flexible metal plates hanging off the end of the conveyor. Yellow elements are fiberglass laminate springs; the motor is not shown.

In part I of this series (*DE* April 2008, p. 16), we explained the concept that every structure has natural frequencies of vibration (eigenvalues) and that these natural frequencies have specific deformation shapes (eigenmodes or normal modes). We also took a swipe at how one would use this information in the structural design world by noting that excitation frequencies outside of a structure's first couple of eigenvalues means it will behave statically stable. We now want to expand upon this theme and demonstrate how this simple form of analysis can be leveraged to uncover how your structure might behave under dynamic loading.

THE DOMINATORS: MODES WITH MASS

An interesting fact about normal modes analysis is that we can associate a percentage of the structure's mass to each mode. With enough modes, you get 100 percent of the mass of the structure, though for complex structures this can mean hundreds of modes. The common thought is that if you capture 90 percent of the mass of the structure that will be good enough. For now, we'll start classically and then show what this concept means in a real-world engineering situation.

We use the supported beam because it is simple to visualize, simple to formulate, and best of all, simple to draw on a white board. In Figure 1 (*next page*), we show a quick example of the first three modes of a simple supported beam along with the percentage of mass associated with the mode. The first mode dominates with 82 percent of the mass of the beam swinging up and down. The second and third modes contribute a little bit of mass but nothing like what we saw in the first mode.

All of these modes operate in one direction. In the real world, the mass fraction is associated with all six degrees of freedom within a particular natural frequency. What this means is that if we excite this first mode in the vertical direction (the direction of the mass fraction), then the structure will move with 82 percent of its mass behind this mode. If we think like Newton and realize that F=m*a, then we can visualize the dominance of this mode and the huge forces that can be generated at resonance.

PEA POD TRANSPORT

Let's leverage this information in a couple of typical engineering problems. Manufacturers use vibrating conveyors to move materials ranging from pea pods to lumps of coal. One such vibrating conveyor is shown above. It moves pea pods within a food-processing plant using a vibratory motor that creates a sinusoidally varying force that is aligned down the axis of the conveyor (y-axis). This force causes the conveyor to swing forward and up on its fiberglass laminate springs.

When operating at its resonant frequency, the conveyor tosses the pea pods forward and upward in a gentle swinging fashion. The material transport rate is determined by its operating frequency and the length and angle of the fiberglass spring laths.

A fundamental problem with this type of conveyor is that during startup as the vibratory motors spin up to speed, nonoperating modes get excited, often causing the conveyor to tear itself apart before it reaches the target operating frequency. Our eigen analysis of the conveyor shows that it has to pass through three modes before reaching its operating frequency at 18Hz. Table 1 shows a brief summary of the data

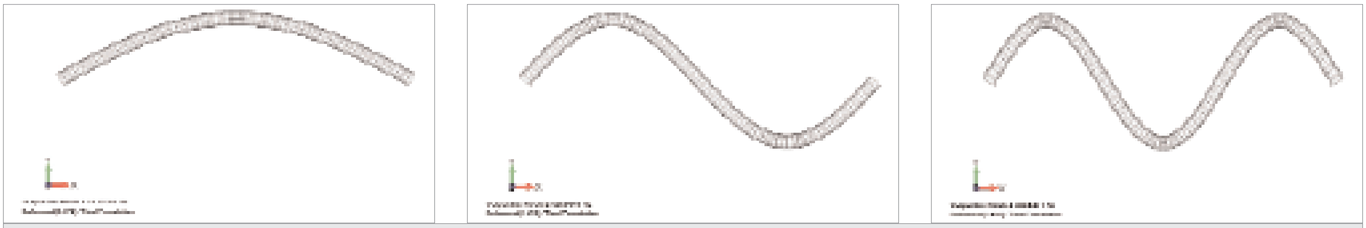


Figure 1: This illustrates a vibration analysis of a simply supported beam. From left, the normal mode shows a specific deformation shape where 82 percent of the mass of the beam swings up and down; the second and third modes of the beam only occur at much higher frequencies. In the second, only 10 percent of the mass deforms this way, and in the third, only 3 percent. These modes have very little effect on the overall behavior.

harvested from this analysis.

Along with this hard data we have the shapes of these four modes shown in Figure 2. We now have a complete picture of the dynamic mechanical behavior of the conveyor system.

Although the vibratory motor has to drive through three modes to reach the target frequency of 18Hz, it has three things in its favor: the applied force is along the y-axis; the mass fractions of the first three modes are small (less than 6 percent); and the dominant directions of the first three modes are not aligned with the forcing function. Thus, with a basic eigen analysis, one can approve the design of a very complex engineering system.

MAKING YOUR RIDE AS SMOOTH AS SILK

Ever wonder what makes a quiet ride in a motor vehicle? It has to do with avoiding modes that might be driven to resonance; that is to say, keeping the structure dynamically static in its mechanical behavior.

In an analysis of a modern motor home, imagine the FEA model is highly idealized using beam elements for the small structural tubes, plate elements for the main longitudinal beams, and lots of mass elements to represent the engine, air conditioners, water and diesel tanks, and passengers. After a normal modes analysis, we have 45 modes ranging from 2.3Hz to 15Hz.

Trying to figure out which mode will cause trouble is essentially impossible without knowing something about the mass participation within each mode. To help us sort through this mess, we can graphically show the mass participation sums for the *x*, *y*, and *z* directions.

Ride smoothness in many cases is just the "hop" in the structure or the bounce in the *y*-direction. The biggest bounce of interest occurs at mode 21 where the mass participation jumps from around 23 percent to 43 percent (20 percent of the mass is moving upward at mode 21). If we investigate this mode a little deeper, we will find out that the entire coach frame is bucking upward at a frequency of around 10Hz. We now have a pretty good picture of what to avoid — anything around 10Hz.

Mode	Eigenvalue	Mass Fraction	Dominant Direction of Mass Fraction
1	3.5Hz	5%	x-axis
2	13Hz	5.8%	z-axis
3	16Hz	1 %	y-axis rotation
4	18Hz	60%	y-axis

Table 1: Summary of Eigen Analysis Results

Luckily, standard road-noise rarely exceeds anything higher than 5Hz. Given our current knowledge of the mode behavior and its mass participation fractions, we are in good shape for a smooth and stable ride.

DESIGNING THE STIFFER STRUCTURE

One of the realities of a normal modes analysis is that you don't get any information about the magnitude (deformation or stress) of the actual response. This is due to the fact that you are not applying a load to the structure. While this poses some limitations, we can also use something called the strain energy density to estimate where the structure is the most flexible or the "weakest."

The mode shape of the structure represents the permissible deformed shape, which directly correlates to the strain energy pattern. Elements with large values of strain are those that most directly affect the natural frequency of that mode. If you can lower the strain energy, you'll increase that frequency.

Figure 3 (*bottom page 66, left*) shows an electronics enclosure that is attached to a couple of brackets. The strain energy density for the first mode is contoured over the brackets. In this design, the brackets are bolted onto the C-channel used as the attachment point to the building. The design goal is to survive a rather severe earthquake (GR-63-Core Zone 4 specification).

To simulate the earthquake the structure is shaken in all three axes. The first mode at 7Hz has 45 percent of the mass swinging back and forth in the Z-direction as correlated by the high strain energy shown in

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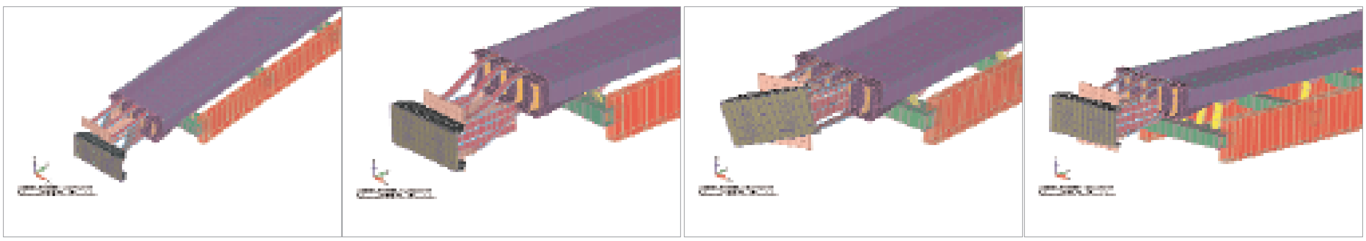

Figure 2: Four vibrational modes of conveyor belt pictured from the left as it ramps up in frequency to final operating value. Mode 1 shape at 3.5Hz has a mass fraction of 5% along the *x* axis; Mode 2 shape at 13Hz has a mass fraction of 5.8% along the *z* axis; Mode 3 shape at 16Hz has a mass fraction of 1% in a *y* axis rotation; and the Mode 4 shape at 18Hz has a mass fraction of 60% along the *y* axis.

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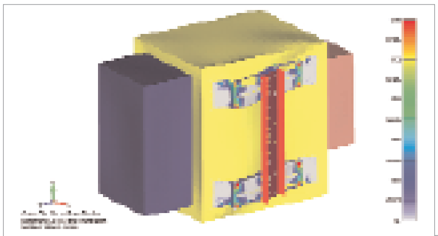

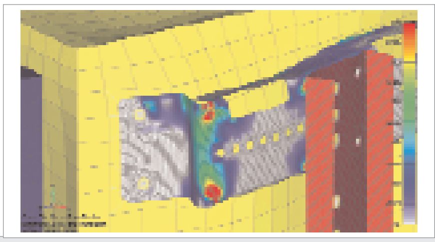

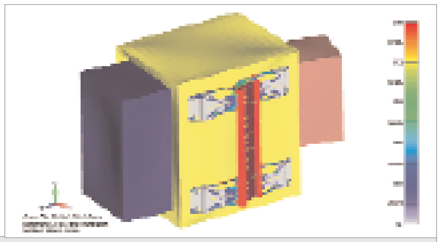

Figure 3: Left) Strain energy density of initial design of electronics housing mounting brackets indicating areas most likely to affect natural frequency of first mode; Center) Bracket subjected to simulated earthquake (GR-63-Core Zone 4 spec) showing high strain energy of first mode at 7Hz at flex points of bracket; Right) Revised bracket design with capped ends displays much lower strain energy and pushes first natural frequency to 10Hz.

Figure 3 (*above center*) at the flex points of the bracket. To improve this design, we only need to address the high-strain energy locations. This is done by capping the ends of the bracket. With the bracket reinforced, the strain energy is reduced (*as shown in Figure 3, above right*) and the frequency jumps to 10Hz.

ADVANCED MODAL ANALYSIS CHECKLIST

Don't panic when you have eigenvalues right on top of your operating frequencies. Only natural frequencies with significant mass participation factors are important.

Eigenmodes have directions as do their mass participation fractions. Investigate these directions and see if they correspond to your forcing-function direction. If they don't (let's say they're orthogonal), then the structure will remain dynamically stable.

If you need to stiffen up your structure, look at the modes where the mass participation is high and then investigate their strain energy density. Modify your structure to lower the strain energy in high-energy sections and you'll see a significant increase in your eigenvalues.

Be methodical and look at your structure from all directions. The secret to making a dynamically stable structure is to tie everything together: eigenvalue (the mode frequency); mode direction (i.e., mode shape); mass participation fraction; mass participation direction; and strain energy density. If you remain cognizant of all of these factors, you will have a good degree of success in not being surprised with aberrant or disastrous dynamic behavior in your structure. ■

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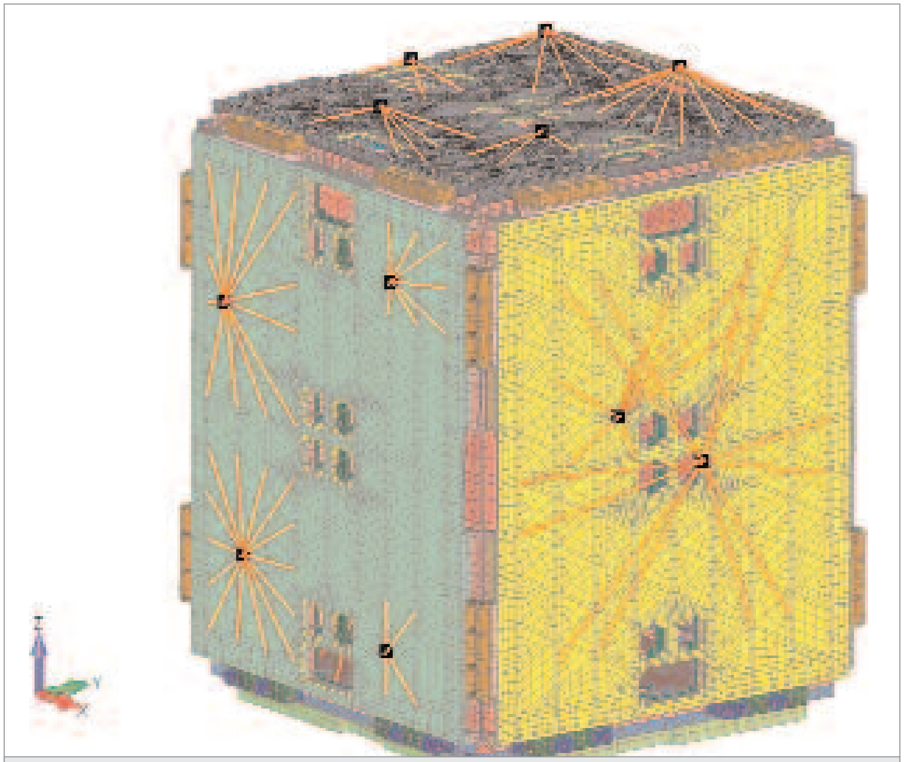
Linear Dynamics for Everyone

> Part 3: Extracting real quantitative data to anticipate everything from earthquakes to rocket launches.

BY GEORGE LAIRD



This is a satellite FEA model showing instrumentation attachment points (black squares) for idealized mass elements as defined by a center of gravity connected by rigid links (orange lines). The model is driven with an input power spectral density (PSD) function.

If you've kept up with this series of articles, you now know more about the dynamic behavior of structures than 95 percent of your peer group within the design and engineering world. And after reading about how vibration analysis reveals key information about structural behavior (see *DE*, April and May 2008), the terms "natural frequencies, normal mode shapes, mass participation factors, and strain energy" have become integral to your vocabulary.

Up to this point the discussion has centered on qualitative terms about the mechanical response of structures due to dynamic loading. In this last part, we'll show how to extract real quantitative data (i.e., displacements and stresses) from a simple normal-modes analysis.

DOING IT ON THE CHEAP

The dynamic response of a structure is derived from its individual normal modes. If you hit your structure, its dynamic response is formed by the summation of its individual modes. Mathematically, we know that each one of our normal modes has a frequency, a mode shape, and a bit of mass associated with that shape (a mass participation factor). All of this data is derived from the basic equation of motion:

$$ma+kx = 0$$

From this equation, the standard linear dynamics solution can be derived as:

$$v = \sqrt{K/m}$$

where v is the frequency or eigenvalue of the system. Since no forces are involved in this equation we can't have any real displacements or stresses.

If we want real data, we need real forces as in: ($ma+kx = F$).

The brute-force approach is to solve the model in the time domain. A time-based displacement, force, or acceleration load is inserted into the model and then the computer makes a few thousand solves. At some later date, we then wade through piles of output data (remember, we are doing a complete solve at each time step) to figure out what went wrong, when, and where. This can be a daunting task and is often just plain impractical.

If the loads are frequency-based (displacement, force, or acceleration as a function of frequency), then the door is wide open to all sorts of very numerically efficient solution strategies. First you perform a normal modes analysis and then apply a

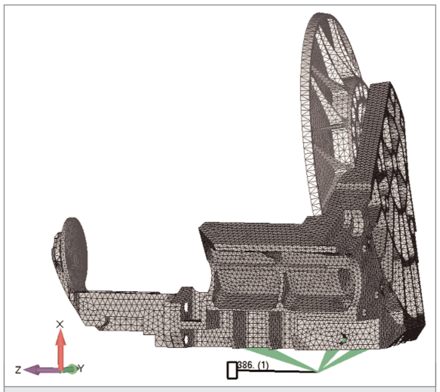


Figure 1: This is an image of an advanced optical platform for an airborne system placed on a virtual shaker table.

frequency-based load. A linear solve is made only at each normal mode or eigenvalue. The resulting displacements and stresses can be viewed individually or summed in some fashion to arrive at a combined damage response to the loading. The technique is extremely useful because you have reduced the number of solves from thousands to just a handful. Although we can't use this technique for every type of structure (linear behavior only), when you can use it, you have the ability to quickly gain insight into the dynamic behavior of the structure on the cheap (minutes versus days).

MODAL FREQUENCY SWEEP (MFS)

Frequency-based loads are more common than you might think. Our previous example was of a vibratory conveyor. Since that was rather straightforward, let's look at something a bit more complex.

Figure 1 shows a high-precision optical-mirror assembly that will be attached to an airplane or helicopter. Airborne structures provide a vibration-rich environment with their high-speed gearboxes, jet turbines, rotating blades, etc. To assess the robustness of the design, you can perform a virtual shaker-table experiment. Our desired output is the deflection response at the focal point of the mirror under severe vibration.

Figure 3a: A rocket launch and other chaotic events create vibration (acceleration) spectra that are best idealized in a statistical sense via power spectral density functions.

Without having to build the mirror, you can input a sin sweep of 1g from 200 to 3000Hz to the model, and see what gets amplified or harmonically driven in the structure. Figures 2a and 2b show the output graph of acceleration and displacement as a function of frequency for the sin sweep.

The mirror system has eigenvalues at 710, 1292, 1570, 1996Hz, etc. But as shown in Figures 2a and 2b, only the mode at 710 Hz creates any real sympathetic accelerations and displacements. This is logical since this mode is aligned with the forcing function and has a mass participation factor of 35 percent in the direction of the forcing function. In a way, we knew what the results would be before we ran the analysis. But now we have real numbers.

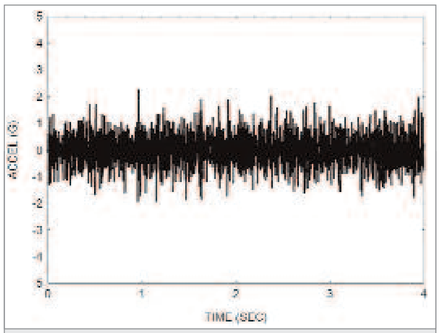


Figure 3b: The rocket acceleration measurements are converted into a power spectral density function with units of acceleration squared against frequency. The applied load is based on frequency.

POWER SPECTRAL DENSITY (PSD) ANALYSIS

Satellites are expensive and failure is even more expensive. During launch (*Figure 3a*) they get pounded by a broad and chaotic spectrum of vibrations (accelerations) from the rocket motor, stage separation, acoustical noise, etc. No single acceleration frequency dominates and there are multiple layers of noisy events that occur randomly.

To numerically model such loading, a statistical approach is used where acceleration measurements are converted into a power spectral density (PSD) function with units of acceleration squared against frequency (*Figure 3b*). Once again, we have a

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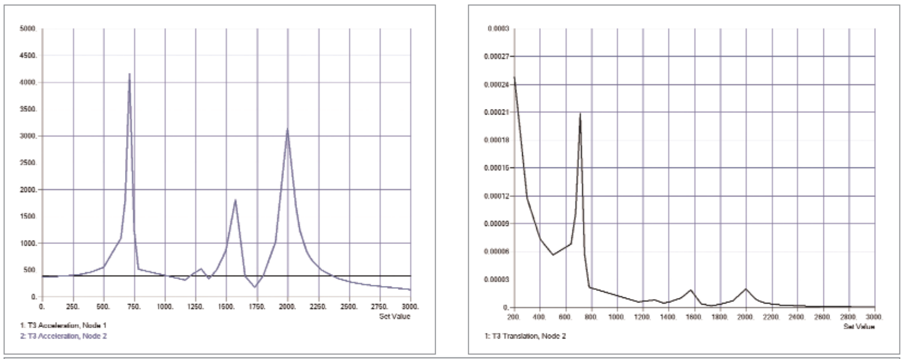
ANALYSIS CHECKLIST

1) If you review the normal mode response, its modes and its mass participation factors, you might not need these more advanced techniques.
2) If you go ahead with this analysis, convert all loads into the frequency domain.
3) Results from modal frequency analyses are in the frequency domain. They provide a virtual snap shot of your structure operating at steady state at the forcing frequency of interest.
4) Results from power spectral density analyses are averages of the forced response. They are estimates of the highest possible displacements and stresses.



Figures 2a and 2b: Virtual shaker-table results of acceleration and displacement at the focal point of the large airborne-mounted mirror showing acceleration in cm per second on the *y*-axis of 2a and displacment in cm on the *y*-axis in 2b.

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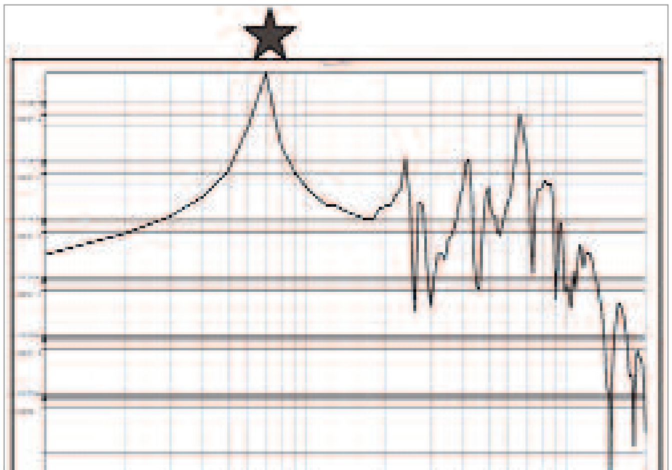

Figure 4: The output spectral density plot of the satellite analysis indicates the location of the first normal mode with the star marking 70Hz. The chart shows frequency on the *x* axis and PSD (g-2/Hz) on the *y* axis.

loading curve where the applied load is based on frequency.

The statistical nature of this calculation comes from the conversion of the acceleration time history data to the PSD function. The PSD amplitudes are actually root mean square (RMS) acceleration values that are fitted to a standard statistical distribution where the mean value is zero and what is plotted is the standard deviation versus frequency. As non-intuitive as this may sound, it is a very effective way to convert chaotic, random noise into a numerically useful load-function.

Another unique aspect of the PSD analysis is that all of the modes of the structure are assumed to be vibrating or excited by the PSD function simultaneously. Somewhat like a bell being rung, the output response is a summation of the amplitudes of all of the frequencies of the structure within the range of interest.

As an example, an FEA model of a satellite (*see page 18*) has various instrument payloads represented as mass elements. These payloads are attached to the main structure of the model with rigid links. In many cases, the utility of a PSD analysis is to determine the transfer function of the structure or how the satellite frame will transmit acceleration into the instrumentation packages.

The resulting transfer function is just another PSD. We then use this output to perform a more detailed PSD analysis on the instrumentation package to determine whether it will survive launch.

The dominant mode of the satellite is around 70Hz (*Figure 4*) and the output PSD function reflects this fact with a huge spike at this frequency (marked with a star). If we knew that our instrumentation package was susceptible to frequencies at 70Hz, our design solution would be to develop a stiffer satellite frame that would not have a dominant normal mode at 70Hz. Even without the PSD analysis, a clear understanding of the normal mode frequencies, their mode shapes, and corresponding mass-participation factors allows you to make valid predictions.

Now you have the concepts, the vocabulary, and the big picture for operating in the world of linear dynamics. A simple modal analysis can put you well on the way to successfully meeting your structural engineering challenges. Why not give it a try? ■

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